

Various Reliability measure of an ATM (Automatic teller machine) due to abnormal weather conditions with inspection and human error

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Abstract

The main purpose of this paper is to analyse the various reliability measure of an ATM (Automatic Teller Machine) due to abnormal weather condition with inspection and human error. The operation, Inspection, replacement and repair of the unit are stopped in abnormal weather as a precautionary measure to avoid excessive damage to the system. Failure rates, repair rates, inspection rates and rate of change of weather condition follow Weibull distribution. Various reliability and economic measures of this system effectiveness are obtained by using Semi Markov process and regenerative point technique. The graphical behaviour of MTSF and profit due to abnormal weather rate has been shown to a particular case.

Keywords weather condition, inspection, human error, Laplace transform, Laplace–Stieltjes Transforms. Regenerative point technique

1-Introduction.

A reliability model of a single -unit repairable system is analysed considering two weather conditions normal and abnormal. For this, a reliability model is developed in which unit may fail totally either directly from normal mode or via partial failure. A single repair facility is available who plays the dual role of inspection and repair. The totally failed unit is first inspected by the server to examine the feasibility of its repair. Therefore, common man prefers single- unit systems due to their affordability and inherent reliability. Many authors have studied single-unit systems under different sets of assumptions on failure and repair policies. A. Kumar & Ajeeta Singh (2021) measures reliability and its characteristic of a stochastic model of an electronic system ATM (Automated Teller Machine) by using internal and external repair policy. Reliability model of a redundant system of two non-identical units is developed by Ashok Kumar, D. Pawar & S.C Malik (2020) which operates normal/abnormal weather conditions. Barak M.S., Neeraj, and Kumari S (2018) analysed the Profit of a two-unit system called the standby system that is working under different weather conditions in an inspection facility. Barzkar, A., Najafzadeh, M. & Homaei, F. (2022) Formulated values of SPEI for various climates by three robust Artificial Intelligence (AI) models: Gene Expression Programming (GEP), Model Tree (MT), and Multivariate Adaptive Regression Spline (MARS).C. Chen, J. Wang and D. Ton (2017) Proposed an integrated solution in the form of a decision support tool to achieve the goal. C. Guo, C. Ye, Y. Ding and P. Wang (2020) developed a fragile model to evaluate the nodal SCF probability considering the insulation aging of equipment and extreme weather condition. C. Wang, Y. Hou, Z. Qin, C. Peng and H. Zhou (2015) proposed a method to establish an optimal dynamic coordinated condition-based maintenance strategy that considers harsh external conditions. Deswal S. and Malik S. C. [2015] obtained reliability measures of a system of two non-identical units operating under normal and abnormal weather conditions in steady state using semi-Markov process and regenerative point technique. D. Chakraborty, S. D. Nair and M. Mukherjee (2023) investigated the rain-attenuation characteristics of terahertz signal in tropical climate condition for different hydrometeor properties including rain-rates and drop-size distribution. The effect

of tropical thunderstorm has been uniquely incorporated by the authors in the present study along with atmospheric humidity and temperature-based fluctuations. F. Mujjuni, T. R. Betts and R. E. Blanchard (2023) reports an increase in the frequency, intensity, and duration of weather-related power outages. Several studies have proposed approaches for evaluating and enhancing power system resilience. Gahlot, Monika; Singh, V.V.; Ayagi, Hamisu Ismial; Goel, C.K. (2018) studied the reliability measures of a complex system consisting two subsystems. Hassan Iqbal, Solomon Tesfamariam, Husnain Haider & Rehan Sadiq (2017) conduct a state-of-the-art review of maintenance policies of O&G pipelines to investigate their advantages, limitations, and associated implementation issues. Maintenance policies can be categorised into corrective, preventive, predictive and proactive. Himani Sharma (2017) Analysed the comprehensive reliability in terms of clearness index for power production of HIT, amorphous silicon & multi-crystalline silicon (m-cSi) technologies. The energy estimation of these three technologies is done based on regression, and deviation in the measured and estimated values is also reported. Kumar, A., Saini, M. (2018) launches two stochastic models of a single-unit system with the notion of degradation and abnormal environment. In both models, system becomes degraded after repair while preventive maintenance of the original or degraded unit is perfect. Kumar A, Saini M (2016) analysed the effect of abnormal environmental conditions on various reliability measures, for this purpose, a stochastic model is developed for single-unit systems by using semi-Markov process. Kumar A, Saini M (2016) analysed the impact of abnormal weather conditions on various reliability measures of a repairable system of single-unit. Koop, S.H.A., van Leeuwen, C.J. (2015) Provides a three-step revision of the City Blueprint Framework (CBF) based on data of 45 municipalities and regions in 27 countries. Puspendu Ghosh & Mala De (2022) reviews previous work on distribution system resilience, concentrating on electrical network protection in the face of extreme events. Firstly, it analyses confounding terminology used in power systems resilience studies, such as definitions, resilience vs reliability, and resilience curve. Saini, M., & Kumar, A. (2021) study covenants with stochastic investigation of an integrated hardware-software system considering hardware failure, software up-gradation upon failure, precautionary maintenance (PM) after a pre-determined process time, maximum repair time of hardware and different weather conditions. Sonal, S. K. Sahu, D. Ghosh and D. K. Mohanta (2019) illustrates a three-component Markov modelling consisting of OH line/UG cable, sensor and a distributed generation (DG). DGs can make the microgrid self-sustained at the time of power loss from the utility grid. During adverse weather condition DG is used as an emergency shelter to keep the distribution system intact. Shan, Xiaofang; Wang, Peng; Lu, Weizhen (2017) presents a novel study to modify these assumptions, and further analyze the influence of changeable outside temperature on the reliability of heating networks based on the state-space method. S. Ma, B. Chen and Z. Wang (2018) proposed an optimal hardening strategy to enhance the resilience of power distribution networks to protect against extreme weather events. Different grid hardening techniques are considered, such as upgrading poles and vegetation management. Zamani Gargari, Milad; Ghaffarpour, Reza (2020) contributes a framework to assess the reliability of energy hub based on its elements and input energy carriers. Loss of load expectation (LOLE), loss of load probability (LOLP) and expected energy not supplied (EENS) have been employed to determine the reliability of the multi-energy carrier system in two different weather condition. Vijay Vir Singh, Praveen Kumar Poonia, Ameer Hassan Abdullahi (2020) presents the study of reliability measures of a complex system consisting of two subsystems, subsystem-1, and subsystem-2, in a series configuration with switching device. Y. Wu, T. Fan and T. Huang (2020) evaluated weather condition models for the probability of component failure and the pre-arranged maintenance are constructed based on the degree of influence of the main weather elements on them.

Notation

O : Operative state

E : Set of regenerative states for each model

$f_1(t) / f_2(t) / f(t)$:	pdf/cdf of failure rate from normal mode to partial failure
$F_1(t) / F_2(t) / F(t)$:	mode /partial failure mode to total failure mode/normal mode to total failure mode.
$z(t)/Z(t)$:	p.d.f./c.d.f. of time to change of weather conditions form normal to abnormal
$z_1(t) / Z_1(t)$:	abnormal to normal
p / q	:	Probability that repair is not feasible/feasible.
$g(t) / G(t)$:	pdf/cdf of repair times of completely failed unit
$h(t) / H(t)$:	pdf/cdf of inspection time
O / PF	:	Unit is operative and in normal mode/unit is partially failed
$\overline{O} / \overline{PF}$:	Unit is good / partially failed but not working due to abnormal weather
FU_i / FU_r	:	Unit is totally failed and under inspection/under repair.
$\overline{FW_r} / \overline{FW_i}$:	Unit is completely failed and waiting for repair/ inspection due to abnormal weather
FH	:	Unit is failed due to human error
$q_{ij}(t) / Q_{ij}(t)$:	pdf and cdf of direct transition time from a regenerative state i to a regenerative state j or a failed state j without visiting any other regenerative state in $(0,t]$
$\phi_i(t)$:	cdf of first passage time from regenerative state i to a failed state.
$A_i(t)$:	Probability that the system is up at epoch
$B_i(t)$:	Probability that server is busy in the system at instant
$N_i(t)$:	Expected number of visits by the server
$M_i(t)$:	Probability that system initially in regenerative state Si remains up till time ' t ' without making any transition to any other regenerative state or returning itself through one or more non-regenerative states
$W_i(t)$:	Probability that the server is busy in the state Si up to time t without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative state.
m_{ij}	:	Contribution to mean sojourn time in state Si when system transits directly to state S_j so that $\mu_i = \sum_i m_{ij}$, where $m_{ij} = \int q_{ij}(t)dt = \int dQ_{ij}(t)dt = -\left[\frac{d}{ds}(Q_{ij}^*(s))\right]_{s=0}$ and μ_i is the mean sojourn time in state S_i .
\otimes / \odot	:	Symbols for Stieltjes convolution / Laplace convolution

$\square / *$: Symbols for Laplace Stieltjes transform/Laplace transform

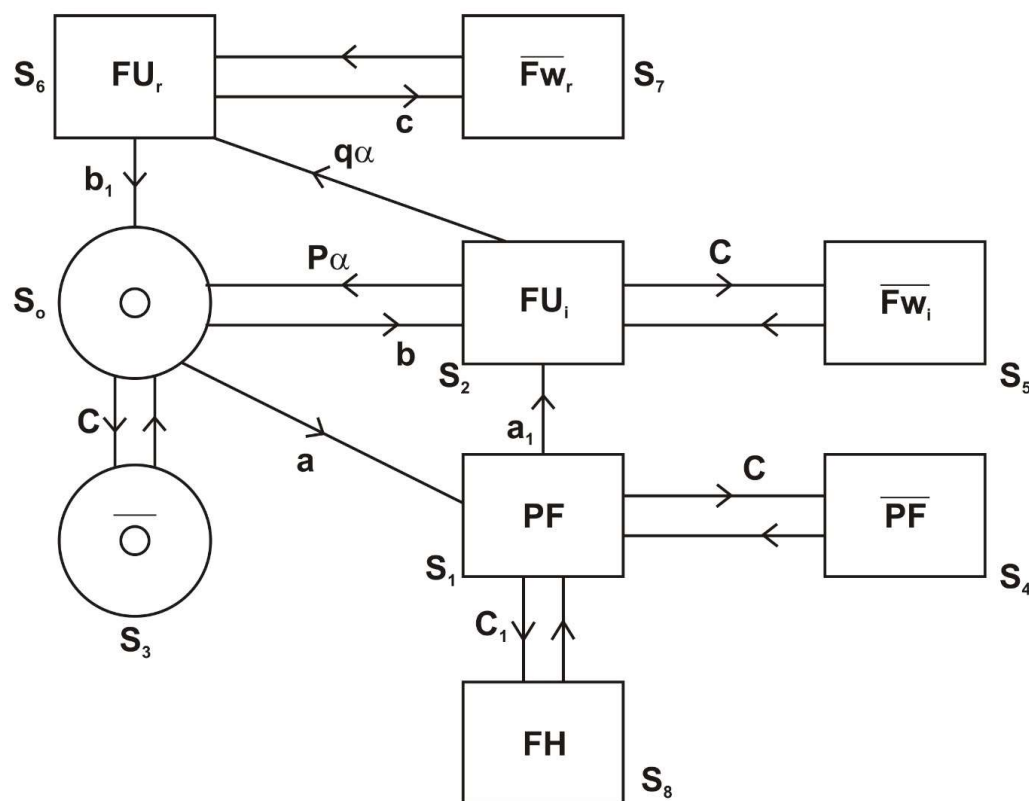
The possible states of system models are shown in following table-1:

S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
O	PF	FU_i	\bar{O}	\bar{PF}	\bar{FW}_i	FU_r	\bar{FW}_i	FH

All the transition states of the model are regenerative

Figure-1

State Transition Diagram



3. transitions probabilities:

Here epochs of entry into any of the states $S_i \in E$ are regenerative point. Let $T_0 (\equiv 0)$, T_1, T_2, \dots denotes the regenerative epochs at which the system enters any state S_i . Let X_n denotes the state visited at epoch T_{n+} i.e. just after the transition at T_n . Then $\{X_n, T_n\}$ considered a Markov-Renewal process with state space E and

$Q_{ij}(t) = P\{X_{n+1} = j, T_{n+1} - T_n \leq t \mid X_n = i\}$ is the semi- Markov kernel over E.

Thus, steady state transition probabilities can be obtained as, $P_{ij} = \lim_{t \rightarrow \infty} Q_{ij}(t)$. By probabilistic arguments, the non-zero elements P_{ij} are

$$\begin{aligned} P_{0,1} &= \int_0^\infty a\eta t^{\eta-1} \exp(-at^\eta) \exp(-bt^\eta) \exp(-ct^\eta) dt = \frac{a}{a+b+c}, & P_{0,2} &= \frac{b}{a+b+c}, & P_{0,3} &= \frac{c}{a+b+c}, \\ P_{1,2} &= \frac{a_1}{a_1+b_1+c}, & P_{1,4} &= \frac{c}{a_1+b_1+c}, & P_{1,8} &= \frac{c_1}{a_1+b_1+c}, & P_{2,5} &= \frac{c}{\alpha+c}, & P_{2,0} &= \frac{p\alpha}{\alpha+c}, & P_{2,6} &= \frac{q\alpha}{\alpha+c}, \\ P_{6,7} &= \frac{c}{b_1+c}, & P_{6,0} &= \frac{b_1}{b_1+c} \text{ and } & P_{3,0} &= P_{4,1} = P_{5,2} = P_{7,6} = P_{8,1} = 1 \end{aligned} \quad (1)$$

It can be easily verified that

$$P_{0,1} + P_{0,2} + P_{0,3} = P_{1,2} + P_{1,4} + P_{1,8} = P_{2,0} + P_{2,5} + P_{2,6} = P_{6,0} + P_{6,7} = 1 \quad (2)$$

4. The mean sojourn time

The mean sojourn times μ_i in the state s_i are given below

$$\mu_0 = m_{01} + m_{02} + m_{03} = \int_0^\infty \exp(-at^\eta) \exp(-bt^\eta) \exp(-ct^\eta) dt = \frac{\left|1 + \frac{1}{\eta}\right|}{(a+b+c)^{\frac{1}{\eta}}} \quad (3)$$

$$\mu_1 = m_{12} + m_{14} + m_{18} = \frac{\left|1 + \frac{1}{\eta}\right|}{(a_1 + c_1 + c)^{\frac{1}{\eta}}},$$

$$\mu_2 = m_{20} + m_{25} + m_{26} = \frac{\left|1 + \frac{1}{\eta}\right|}{(c + \alpha)^{\frac{1}{\eta}}}, \quad (4) \quad \mu_3 = \mu_4 = \mu_5 = \mu_7 = \mu_8 = \frac{\left|1 + \frac{1}{\eta}\right|}{\beta^{\frac{1}{\eta}}}, \quad \mu_6 = \frac{\left|1 + \frac{1}{\eta}\right|}{(b_1 + c)^{\frac{1}{\eta}}}$$

(5)

5. Reliability and Mean time to system failure:

To obtain MTSF we treated failed state as absorbing. Let $\phi_i(t)$ be the cumulative distribution function (cdf) of first passage time from regenerative state i to failed state. then applying the argument used for regenerative process we have the following recursive relation for $\phi_i(t)$.

$$\phi_i(t) = \sum_j Q_{i,j}(t) \otimes \phi_j(t) + \sum_k Q_{i,k}(t) \quad (6)$$

where j is an operative regenerative state to which the given regenerative state i can transit and k is a failed state to which the state i can directly transit. By using we obtain the following relations.

$$\phi_0(t) = Q_{0,1}(t) \otimes \phi_1(t) + Q_{0,3}(t) \otimes \phi_3(t) + Q_{0,2}(t) \quad (7)$$

$$\phi_1(t) = Q_{1,4}(t) \otimes \phi_4(t) + Q_{1,2}(t) + Q_{1,8}(t) \quad (8)$$

$$\phi_3(t) = Q_{3,0}(t) \otimes \phi_0(t) \quad (9)$$

$$\phi_4(t) = Q_{4,1}(t) \otimes \phi_1(t) \quad (10)$$

Taking LST of (7) to (10) and on solving the solution for $\bar{\phi}_0(s)$ can be written as,

$$\bar{\phi}_0(s) = \frac{N_1(s)}{D_1(s)} \quad (11)$$

Where

$$N_1(s) = \bar{Q}_{0,1} \bar{Q}_{1,2} + \bar{Q}_{0,1} \bar{Q}_{1,8} + \bar{Q}_{0,2} \bar{Q}_{1,4} \bar{Q}_{4,1} \quad \text{and}$$

$$D_1(s) = 1 - \bar{Q}_{1,4} \bar{Q}_{4,1} - \bar{Q}_{0,3} \bar{Q}_{3,0} + \bar{Q}_{0,3} \bar{Q}_{3,0} \bar{Q}_{1,4} \bar{Q}_{4,1}$$

using this we have

$$R^*(s) = \frac{1 - \bar{\phi}_0(s)}{s} \quad (12)$$

The reliability $R(t)$ can be obtained by taking inverse Laplace transform of (12). The mean time to system failure is given by

$$MTSF = \lim_{s \rightarrow 0} R^*(s) = \frac{D_1'(0) - N_1'(0)}{D_1(0)} \quad (13) \quad \text{where}$$

$$D_1'(0) - N_1'(0) = \mu_0(1 - p_{14}) + \mu_1 p_{01} + \mu_3 p_{03}(1 - p_{14}) + \mu_4 p_{14} p_{01}$$

And

$$D_1(0) = 1 - p_{14} - p_{03} - p_{03} p_{14}$$

6. Steady state availabilities

Let $A_i(t)$ be availability which regarded as the probability that the system is in upstate at any instant of time t . It is given that system entered into the regenerative state i at time $t = 0$. The recursive relation for $A_i(t)$ is given by,

$$A_i(t) = M_i(t) + \sum_j q_{i,j}(t) \otimes A_j(t) \quad (14)$$

Where j is any successive regenerative state to which the regenerative state i can transit, and $M_i(t)$'s is given by

$$M_0 = \exp(-c + b + a)t^\eta, M_1 = \exp(-c + a_1)t^\eta \quad (15)$$

By above equations and probabilistic argument $A_i(t)$ are obtain as

$$A_0(t) = M_0(t) + q_{0,1}(t) \otimes A_1(t) + q_{0,2}(t) \otimes A_2(t) + q_{0,3}(t) \otimes A_3(t) \quad (16)$$

$$A_1(t) = M_1(t) + q_{1,2}(t) \otimes A_2(t) + q_{1,4}(t) \otimes A_4(t) + q_{1,8}(t) \otimes A_8(t) \quad (17)$$

$$A_2(t) = q_{2,0}(t) \otimes A_0(t) + q_{2,5}(t) \otimes A_5(t) + q_{2,6}(t) \otimes A_6(t) \quad (18)$$

$$A_3(t) = q_{3,0}(t) \otimes A_0(t), \quad A_4(t) = q_{4,1}(t) \otimes A_1(t), \quad A_5(t) = q_{5,2}(t) \otimes A_2(t), \quad (19)$$

$$A_6(t) = q_{6,0}(t) \otimes A_0(t) + q_{6,7}(t) \otimes A_7(t), \quad A_7(t) = q_{7,6}(t) \otimes A_6(t), \quad A_8(t) = q_{8,1}(t) \otimes A_1(t), \quad (20)$$

Taking Laplace Transform of above relations and solve for $A^*(s)$

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)} \quad (21)$$

Where

$$\begin{aligned} N_2(s) &= M_0^*(1 - q_{2,5}^* q_{5,2}^*)(1 - q_{6,7}^* q_{7,6}^*)(1 - q_{1,4}^* q_{4,1}^* - q_{1,8}^* q_{8,1}^*) + q_{0,1}^* M_1^*(1 - q_{2,5}^* q_{5,2}^*)(1 - q_{6,7}^* q_{7,6}^*) \\ D_2(s) &= (1 - q_{0,3}^* q_{3,0}^*)(1 - q_{2,5}^* q_{5,2}^*)(1 - q_{6,7}^* q_{7,6}^*)(1 - q_{1,4}^* q_{4,1}^* - q_{1,8}^* q_{8,1}^*) - q_{0,1}^* q_{1,2}^* q_{2,0}^* (1 - q_{6,7}^* q_{7,6}^*) \\ &\quad - q_{0,1}^* q_{1,2}^* q_{2,6}^* q_{6,0}^* - q_{02}^* q_{2,0}^* (1 - q_{6,7}^* q_{7,6}^*)(1 - q_{1,4}^* q_{4,1}^* - q_{1,8}^* q_{8,1}^*) - q_{02}^* q_{2,6}^* q_{6,0}^* (1 - q_{1,4}^* q_{4,1}^* - q_{1,8}^* q_{8,1}^*) \\ M_2^*(0) &= \mu_2 \end{aligned} \quad M_1^*(0) = \mu_1,$$

The steady state Availability is given by

$$A_0(\infty) = \lim_{s \rightarrow 0} s A_0^*(s) = \frac{N_2(0)}{D_2(0)} \quad (22)$$

Where

$$\begin{aligned} N_2(0) &= (1 - p_{25} p_{52})(1 - p_{67} p_{76}) [\mu_0 (1 - p_{14} p_{41} - p_{18} p_{81}) + p_{01} \mu_1] \\ D_2'(0) &= (\mu_0 + p_{03} \mu_3) [(1 - p_{14} - p_{18})(1 - p_{25})(1 - p_{67})] + (\mu_1 + p_{14} \mu_4 + p_{18} \mu_8) [p_{01} (1 - p_{67})(1 - p_{25})] + \\ &\quad (\mu_2 + p_{25} \mu_5) (1 - p_{67}) [(1 - p_{14} - p_{18}) p_{02} + p_{01} p_{12}] + (\mu_6 + p_{67} \mu_7) [p_{26} \{p_{02} (1 - p_{14} - p_{18}) + p_{01} p_{12}\}] \end{aligned}$$

7. Busy period analysis

Let $B_i(t)$ be the probability that server is busy at any instant time t in repairing the unit and the system entered regenerative state S_i at $t = 0$. By probabilistic argument the following recursive relation are obtain as,

$$B_i(t) = W_i(t) + \sum_j q_{ij}(t) \odot B_j(t) \quad (23)$$

where j is any consecutive regenerative state to which the restore state i can transit and $W_i(t)$'s is given by

$$W_2(t) = \exp(-c + \alpha)t^\eta, W_6 = \exp(-c + b_1)t^\eta \quad (24)$$

By above equation and using Probabilistic argument the recursive relation is given as

$$B_0(t) = q_{0,1}(t) \odot B_1(t) + q_{0,2}(t) \odot B_2(t) + q_{0,3}(t) \odot B_3(t) \quad (25)$$

$$B_1(t) = q_{1,2}(t) \odot B_2(t) + q_{1,4}(t) \odot B_4(t) + q_{1,8}(t) \odot B_8(t) \quad (26)$$

$$B_2(t) = W_2(t) + q_{2,0}(t) \odot B_0(t) + q_{2,5}(t) \odot B_5(t) + q_{2,6}(t) \odot B_6(t) \quad (27)$$

$$B_3(t) = q_{3,0}(t) \odot B_0(t), B_4(t) = q_{4,1}(t) \odot B_1(t), B_5(t) = q_{5,2}(t) \odot B_2(t) \quad (28)$$

$$B_6(t) = W_6(t) + q_{6,0}(t) \odot B_0(t) + q_{6,7}(t) \odot B_7(t), B_7(t) = q_{7,6}(t) \odot B_6(t), B_8(t) = q_{8,1}(t) \odot B_1(t) \quad (29)$$

Taking Laplace-Transformation of (25) to (29) and solution for $B_0^*(s)$ is obtain as,

$$B_0^*(s) = \frac{N_3(s)}{D_3(s)} \quad (30)$$

Where

$$\begin{aligned} N_3(s) &= W_2^* q_{0,1}^* q_{1,2}^* (1 - q_{6,7}^* q_{7,6}^*) + q_{0,1}^* q_{1,2}^* q_{2,6}^* W_6^* + W_2^* q_{0,2}^* (1 - q_{6,7}^* q_{7,6}^*)(1 - q_{1,4}^* q_{4,1}^* - q_{1,8}^* q_{8,1}^*) \\ &\quad + W_6^* q_{0,2}^* q_{2,6}^* (1 - q_{1,4}^* q_{4,1}^* - q_{1,8}^* q_{8,1}^*) \end{aligned}$$

And $D_3(s) = D_2(s)$

The busy period can be determined as

$$B_o(\infty) = \lim_{s \rightarrow 0} sB_o^*(s) = \frac{N_3(0)}{D_3'(0)} \quad (31)$$

Where

$$N_3(0) = W_2(1 - p_{6,7} - p_{7,6})[p_{01}p_{12} + p_{02}(1 - p_{14}p_{41} - p_{18}p_{81})] + W_6p_{26}[p_{01}p_{12} + p_{02}(1 - p_{14}p_{41} - p_{18}p_{81})]$$

And $D_3'(0) = D_2'(0)$

8. Expected Number of Visits by the Server

Let $N_i(t)$ be the expected number of visits by the server and the system get into the restore state i at $t = 0$. The recurrence relation for $N_i(t)$ is given by:

$$N_i(t) = \sum_j Q_{i,j}(t) \otimes [\delta_j + N_j(t)] \quad (32)$$

Where j is any regenerative state to which the given restore state can transits and δ_j is Kronecker delta.

$$N_0(t) = Q_{0,1}(t) \otimes [1 + N_1(t)] + Q_{0,2}(t) \otimes N_2(t) + Q_{0,3}(t) \otimes N_3(t) \quad (33)$$

$$N_1(t) = Q_{1,2}(t) \otimes [1 + N_2(t)] + Q_{1,4}(t) \otimes N_4(t) + Q_{1,8}(t) \otimes N_8(t) \quad (34)$$

$$N_2(t) = Q_{2,0}(t) \otimes N_0(t) + Q_{2,5}(t) \otimes N_5(t) + Q_{2,6}(t) \otimes N_6(t) \quad (35)$$

$$N_3(t) = Q_{3,0}(t) \otimes N_0(t), N_4(t) = Q_{4,1}(t) \otimes N_1(t), N_5(t) = Q_{5,2}(t) \otimes N_2(t) \quad (36)$$

$$N_6(t) = Q_{6,0}(t) \otimes N_0(t) + Q_{6,7}(t) \otimes N_7(t), N_7(t) = Q_{7,6}(t) \otimes N_6(t), N_8(t) = Q_{8,1}(t) \otimes N_1(t) \quad (37)$$

Taking LST of the above relations and solving for $N_0^*(s)$

$$N_0^*(s) = \frac{N_4(s)}{D_4(s)} \quad (38)$$

Where

$$N_4(s) = Q_{0,1}^*(1 - Q_{2,5}^*Q_{5,2}^*)(1 - Q_{6,7}^*Q_{7,6}^*)(1 - Q_{1,4}^*Q_{4,1}^* - Q_{1,8}^*Q_{8,1}^*) + Q_{0,1}^*Q_{1,2}^*(1 - Q_{2,5}^*Q_{5,2}^*)(1 - Q_{6,7}^*Q_{7,6}^*)$$

And $D_4(s) = D_2(s)$

Expected number of visits per unit time is given by

$$N_o(\infty) = \lim_{s \rightarrow 0} sN_o^*(s) = \frac{N_4(0)}{D_4'(0)} \quad (39)$$

Where

$$N_4(0) = (1 - p_{2,5} - p_{5,2})(1 - p_{6,7} - p_{7,6})[p_{01}(1 - p_{14}p_{41} - p_{18}p_{81}) + p_{01}p_{12}]$$

And $D_4'(0) = D_2'(0)$

9. Profit analysis

The expected profit gain by the system is given as

$$P = K_0A_0 - K_1B_0 - K_2N_0 \quad (40)$$

Where

P = Profit per unit time gain by the system.

K_0 = Revenue per unit up time of the system.

K_1 = Cost per unit time for which the server is busy.

K_2 = Cost per visit by the server.

A_0, B_0 and N_0 has been already defined in previous sections.

10. Results and Discussion

For the importance of results and characteristics of MTSF, availability and profit of the system, we assume that failure times of unit, time of change of weather state, inspection time and repair times of the unit are Weibull distributed. Probability density function of Weibull distribution is given by

$$z(t) = c\eta t^{\eta-1} \exp(-ct^\eta), t \geq 0$$

Where η and c are positive constants which is known as shape and scale parameters respectively. by property of Weibull distribution, if $\eta = 1$, it convert into the exponential distribution and for $\eta = 2$ it convert to Rayleigh distribution. The probability density function (PDF) for time of change of weather conditions, repair time of the unit, failure time of the unit and inspection time are considered as:

$$z(t) = c\eta t^{\eta-1} \exp(-ct^\eta), \quad z_1(t) = \beta\eta t^{\eta-1} \exp(-\beta t^\eta), \quad g(t) = b_1\eta t^{\eta-1} \exp(-b_1 t^\eta) \quad f_1(t) = a\eta t^{\eta-1} \exp(-at^\eta), \\ f_2(t) = a_1\eta t^{\eta-1} \exp(-a_1 t^\eta) \quad f(t) = b\eta t^{\eta-1} \exp(-bt^\eta) \quad \text{and} \quad h(t) = \alpha\eta t^{\eta-1} \exp(-\alpha t^\eta)$$

For specific values to different parameters and costs, the algebraic outcome for MTSF, availability and profit function are obtained by considering the shape parameter $\eta = 0.5, 1, 2$ for all random variables associated with failure, weather conditions and repair times as shown in table 1,2 and 3.

11. Conclusion

The algebraic outcome of mean time to system failure (MTSF) corresponding to abnormal weather rate (c) and shape parameter (η) are exposed in table 1. Also, the graphical behaviour of MTSF corresponding to abnormal weather rate (c) are exposed in fig 2. From algebraic outcome and graphical behaviour of MTSF it is noticed that MTSF increase to the increase of c . The nature of availability and profit of the model corresponding to abnormal weather rate (c) and shape parameter (η) are exposed in table 2,3 respectively. Also, the graphical behaviour of availability and profit of the model corresponding to abnormal weather rate (c) are exposed in fig 3,4 respectively availability. From algebraic outcome and graphical behaviour of availability and profit it is noticed that availability and profit of the system decreases to the increase of c . Finally, we obtain that to the passes of time the availability and profit of the system decreases.

Table-2 MTSF vs. Abnormal Weather Rate (c)

$\eta = 1$				$\eta = 2$			$\eta = 0.5$		
c ↓	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$
	$b = .02$	$b = .02$	$b = .02$	$b = .02$	$b = .02$	$b = .02$	$b = .02$	$b = .02$	$b = .02$
	$a = .03$	$a = .03$	$a = .03$	$a = .03$	$a = .03$	$a = .03$	$a = .03$	$a = .03$	$a = .03$
	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$
	$\beta = .1$	$\beta = .1$	$\beta = .2$	$\beta = .1$	$\beta = .1$	$\beta = .2$	$\beta = .1$	$\beta = .1$	$\beta = .2$
	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$
	$q = 0.7$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$
	$c_1 = .01$	$c_1 = .01$	$c_1 = .02$	$c_1 = .01$	$c_1 = .01$	$c_1 = .02$	$c_1 = .01$	$c_1 = .01$	$c_1 = .02$

	$\alpha = 2$	$\alpha = 4$	$\alpha = 2$	$\alpha = 2$	$\alpha = 4$	$\alpha = 2$	$\alpha = 2$	$\alpha = 4$	$\alpha = 2$
0.01	33.0000	31.4285	31.5000	7.5032	7.2922	07.2668	0.9989	0.9907	0.9897
0.02	36.0000	34.2856	33.0000	8.8915	8.5582	08.3894	1.0601	1.0041	1.0001
0.03	39.0000	37.1428	34.5000	10.1618	9.8241	09.4233	1.1221	1.0120	1.0020
0.04	42.0000	39.9999	36.0000	11.4911	11.0142	10.4867	1.1372	1.0371	1.0171
0.05	45.0000	42.8571	37.5000	12.7022	12.2296	11.4911	1.1501	1.0500	1.0300
0.06	48.0000	45.7142	39.0000	14.0020	13.3943	12.5249	1.1757	1.0758	1.0458
0.07	51.0000	48.5714	40.5000	15.2131	14.5590	13.4998	1.1915	1.0913	1.0613
0.08	54.0000	51.4285	42.0000	16.4242	15.7237	14.4451	1.2040	1.1043	1.1003
0.09	57.0000	54.2857	43.5000	17.6058	16.8884	15.3903	1.2330	1.1331	1.1131
0.10	60.0000	57.1428	45.0000	18.8169	18.0025	16.3356	1.2669	1.1667	1.1267
0.11	63.0000	59.9999	46.5000	19.9986	19.1166	17.2809	1.3042	1.2040	1.2001
0.12	66.0000	62.8571	48.0000	21.1211	20.2307	18.1966	1.3441	1.2445	1.2045
0.13	69.0000	65.7142	49.5000	22.3027	21.3194	19.1419	1.3872	1.2876	1.2376
0.14	72.0000	68.5714	51.0000	23.4547	22.4082	20.0281	1.4301	1.3328	1.3028
0.15	75.0000	71.4285	52.5000	24.5477	23.4969	20.8848	1.4736	1.3800	1.3200

Table-3 Availability vs. Abnormal Weather Rate (c)

$\eta = 1$				$\eta = 2$			$\eta = 0.5$		
c \downarrow	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$
	$b = .02$	$b = .02$	$b = .02$	$b = .02$	$b = .02$	$b = .02$	$b = .02$	$b = .02$	$b = .02$
	$a = .03$	$a = .03$	$a = .03$	$a = .03$	$a = .03$	$a = .03$	$a = .03$	$a = .03$	$a = .03$
	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$
	$\beta = .1$	$\beta = .1$	$\beta = .2$	$\beta = .1$	$\beta = .1$	$\beta = .2$	$\beta = .1$	$\beta = .1$	$\beta = .2$
	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$
	$q = 0.7$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$
	$c_1 = .01$	$c_1 = .01$	$c_1 = .01$	$c_1 = .01$	$c_1 = .01$	$c_1 = .01$	$c_1 = .01$	$c_1 = .01$	$c_1 = .01$
$\alpha = 2$	$\alpha = 4$	$\alpha = 2$	$\alpha = 2$	$\alpha = 4$	$\alpha = 2$	$\alpha = 2$	$\alpha = 4$	$\alpha = 2$	
0.01	0.8483	0.8546	0.9024	3.5320	3.1809	3.4409	0.1353	0.2570	0.1428
0.02	0.7797	0.7855	0.8621	2.8776	2.7586	3.0534	0.1264	0.2398	0.1398
0.03	0.7213	0.7267	0.8252	2.6936	2.4408	2.7444	0.1167	0.2209	0.1370
0.04	0.6711	0.6761	0.7913	2.2889	2.1948	2.5000	0.1068	0.2018	0.1337
0.05	0.6275	0.6321	0.7601	2.0860	1.9963	2.3054	0.0970	0.1833	0.1299
0.06	0.5891	0.5935	0.7313	1.9093	1.8328	2.1305	0.0879	0.1658	0.1256
0.07	0.5557	0.5593	0.7045	1.7660	1.6967	1.9876	0.0793	0.1498	0.1213
0.08	0.5249	0.5288	0.6797	1.6478	1.5802	1.8669	0.0717	0.1353	0.1166
0.09	0.4978	0.5015	0.6565	1.5414	1.4794	1.7582	0.0648	0.1222	0.1119
0.10	0.4734	0.4769	0.6349	1.4532	1.3925	1.6667	0.0586	0.1106	0.1071
0.11	0.4512	0.4546	0.6147	1.3722	1.3163	1.5826	0.0532	0.1003	0.1024

0.12	0.4310	0.4343	0.5957	1.3008	1.2466	1.5066	0.0483	0.0911	0.0976
0.13	0.4126	0.4157	0.5778	1.2422	1.1856	1.4466	0.0440	0.0831	0.0931
0.14	0.3956	0.3986	0.5610	1.1799	1.1303	1.3781	0.0402	0.0758	0.0887
0.15	0.3800	0.3829	0.5451	1.1267	1.0811	1.3223	0.0368	0.0694	0.0845

Table-4 Profit vs. Abnormal Weather Rate (c)

$\eta = 1$				$\eta = 2$			$\eta = 0.5$		
c ↓	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$	$b_1 = 1$
	$b = .02$	$b = .02$	$b = .02$	$b = .02$	$b = .02$	$b = .02$	$b = .02$	$b = .02$	$b = .02$
	$a = .03$	$a = .03$	$a = .03$	$a = .03$	$a = .03$	$a = .03$	$a = .03$	$a = .03$	$a = .03$
	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$	$a_1 = .05$
	$\beta = .1$	$\beta = .1$	$\beta = .2$	$\beta = .1$	$\beta = .1$	$\beta = .2$	$\beta = .1$	$\beta = .1$	$\beta = .2$
	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$
	$q = 0.7$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$	$p = 0.3$
	$c_1 = .01$	$c_1 = .01$	$c_1 = .01$	$c_1 = .01$	$c_1 = .01$	$c_1 = .01$	$c_1 = .01$	$c_1 = .01$	$c_1 = .01$
	$\alpha = 2$	$\alpha = 4$	$\alpha = 2$	$\alpha = 2$	$\alpha = 4$	$\alpha = 2$	$\alpha = 2$	$\alpha = 4$	$\alpha = 2$
0.01	4225.8	4259.9	4495.2	3646.3	3722.8	3792.7	4580.7	4584.2	4881.1
0.02	3884.0	3915.4	4249.2	3452.2	3466.9	3608.3	4229.2	4234.8	4774.3
0.03	3593.3	3622.4	4110.4	3205.1	3266.9	3457.9	3864.6	3871.9	4649.4
0.04	3343.1	3370.2	3941.6	3058.5	3109.8	3342.3	3505.9	3514.6	4509.6
0.05	3125.5	3150.8	3786.2	2926.9	2975.2	3232.3	3186.6	3175.5	4386.7
0.06	2934.5	2958.3	3642.5	2813.7	2861.2	3138.9	2851.9	2861.8	4198.4
0.07	2765.4	2787.9	3509.5	2707.5	2761.9	3048.2	2567.0	2577.0	4033.0
0.08	2614.8	2633.5	3387.7	2638.6	2673.6	2987.9	2311.3	2321.3	3864.5
0.09	2479.8	2500.0	3270.4	2561.2	2595.3	2920.7	2083.8	2093.5	3695.1
0.10	2357.9	2377.2	3162.6	2495.7	2500.8	2864.1	1882.3	1891.5	3527.5
0.11	2247.6	2265.9	3061.8	2435.4	2462.0	2808.9	1704.0	1712.9	3362.2
0.12	2147.1	2166.6	2967.2	2379.9	2403.8	2757.4	1546.7	1555.1	3201.1
0.13	2055.2	2071.9	2878.3	2323.6	2351.2	2703.1	1407.6	1415.6	3044.8
0.14	1970.8	1986.9	2794.5	2274.6	2301.7	2660.1	1284.6	1292.2	2984.8
0.15	1893.1	1908.6	2715.5	2232.4	2258.1	2617.8	1175.6	1182.7	2750.8

Figure-2

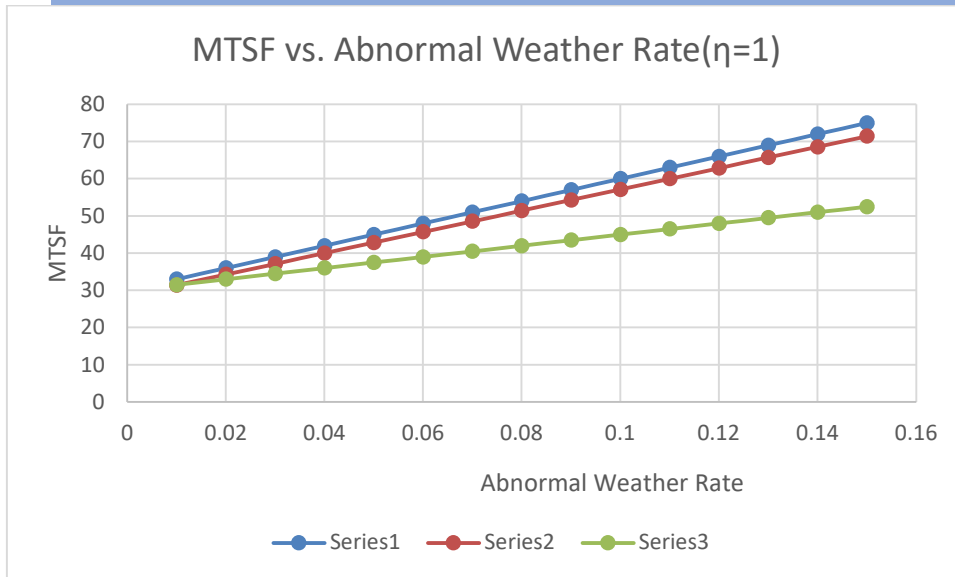


Figure-3

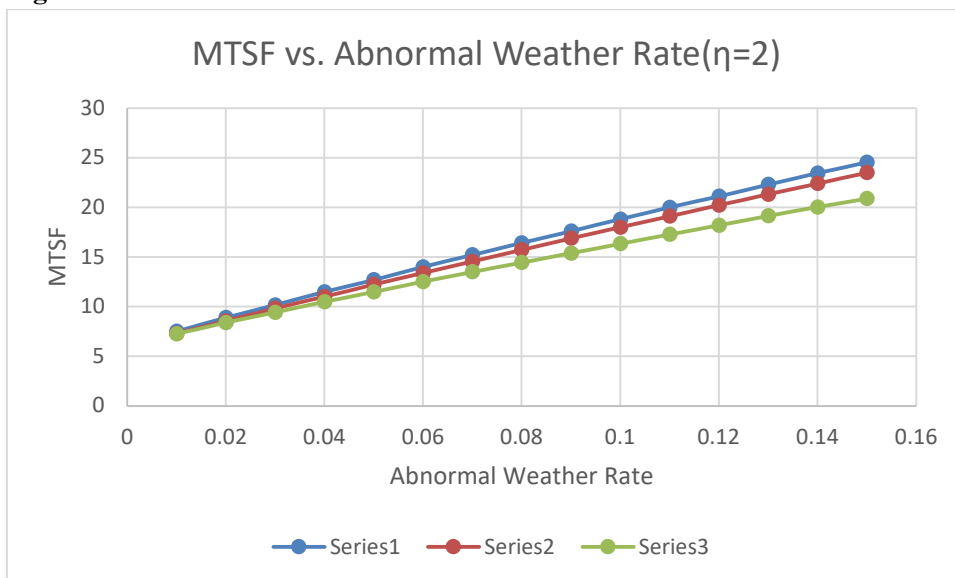


Figure-4

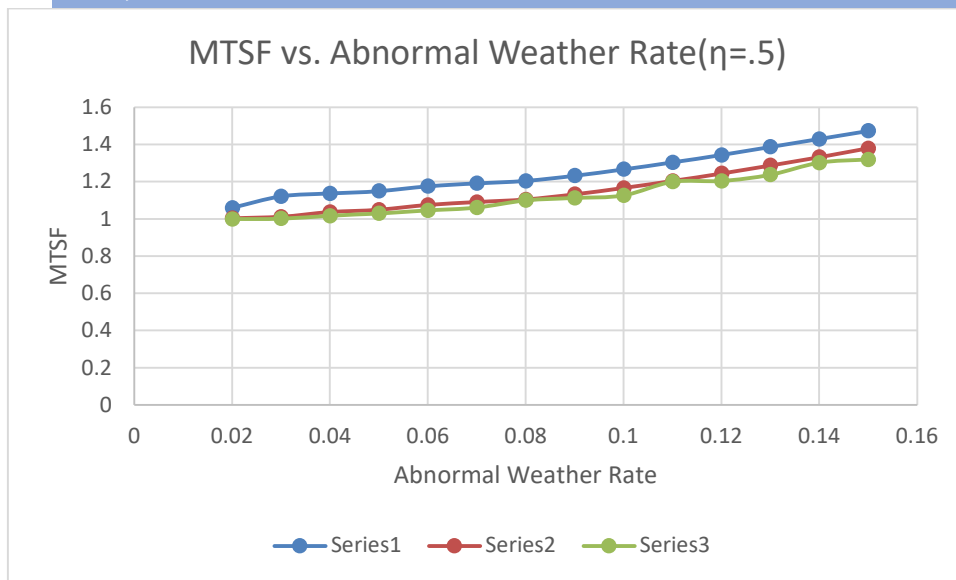


Figure-5

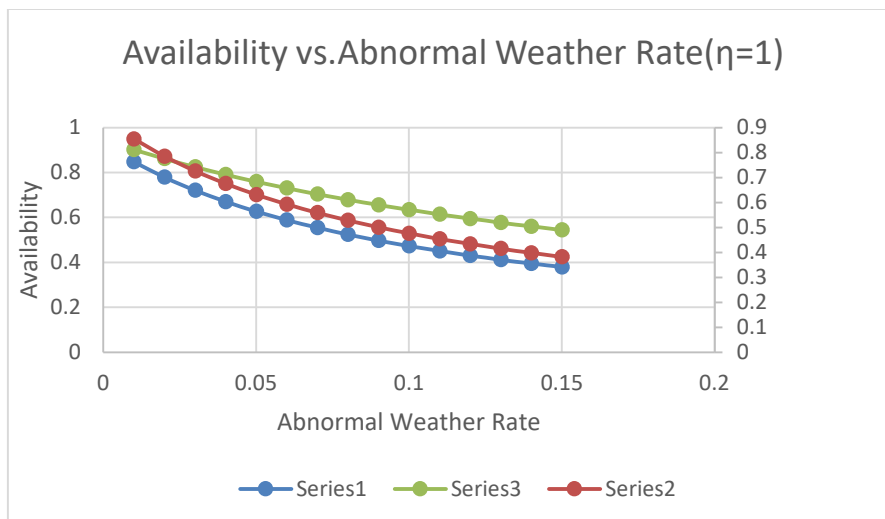


Figure-6

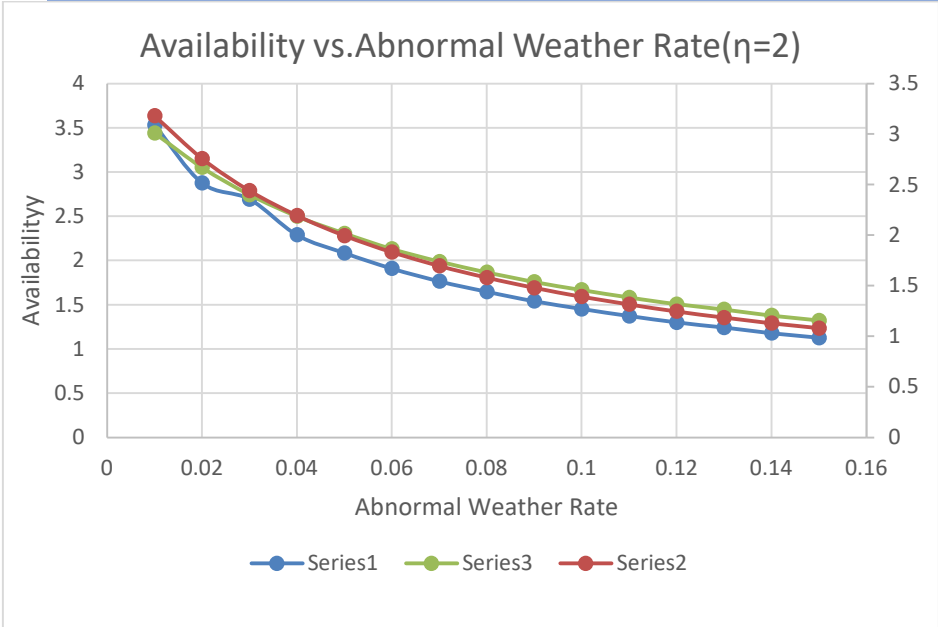


Figure-7

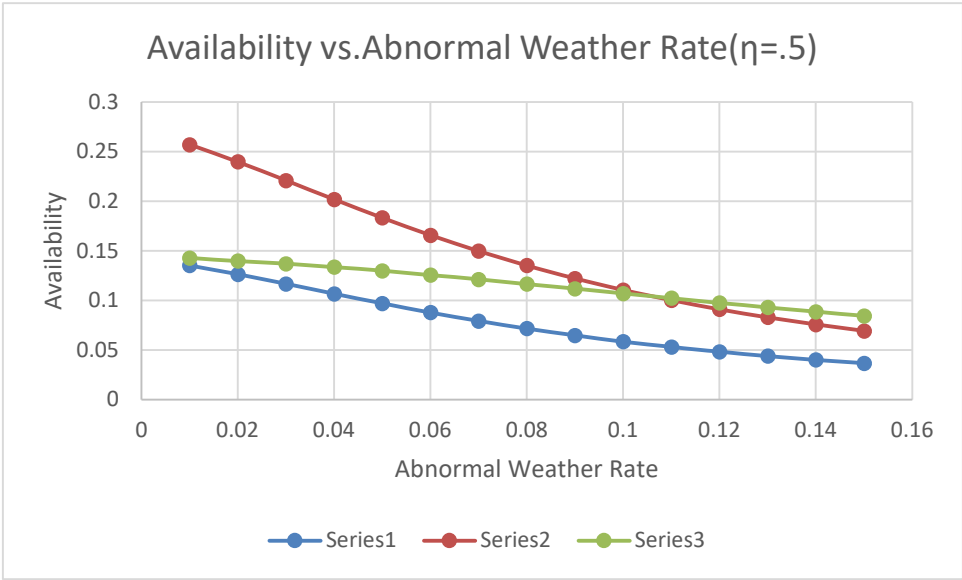


Figure-8

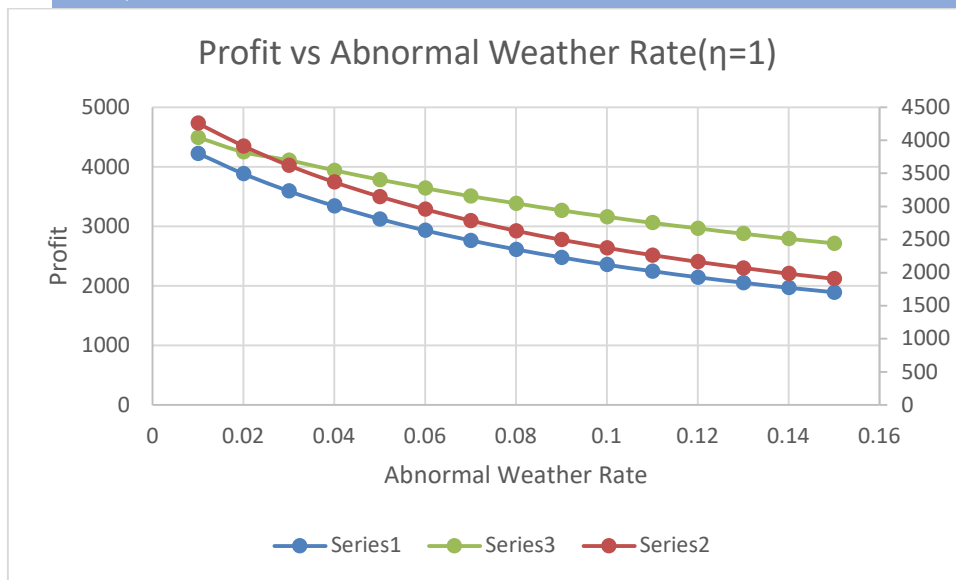


Figure-9

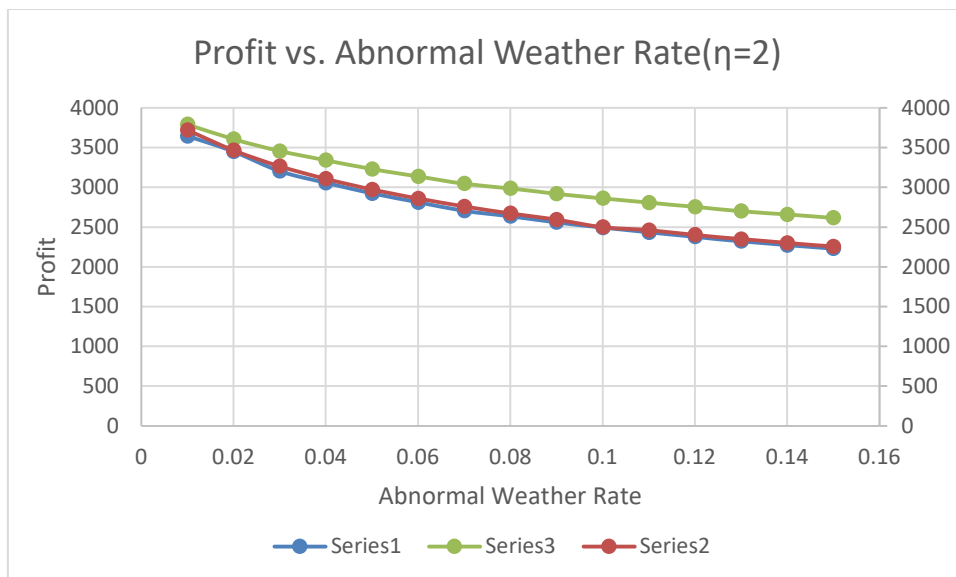
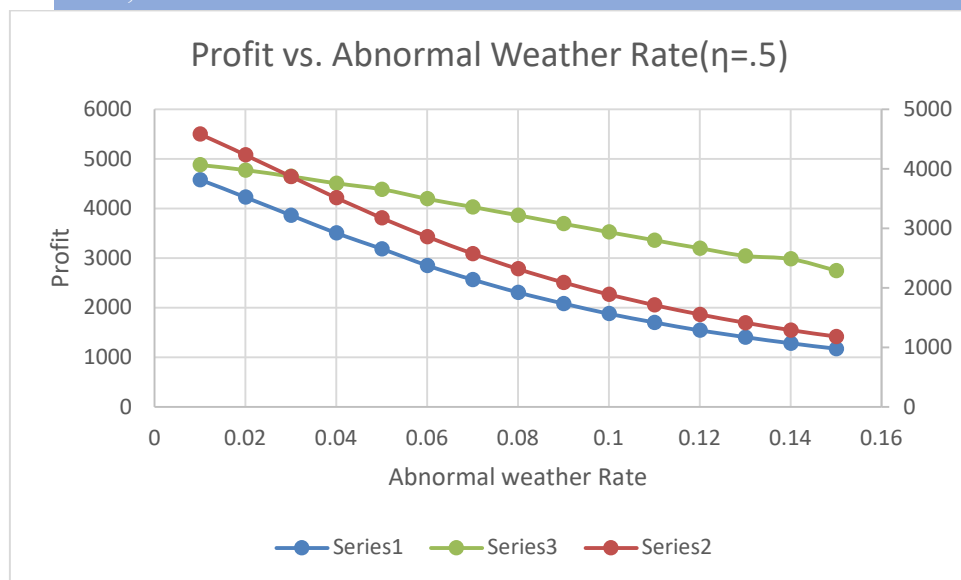


Figure-10



References-

1. Kumar & Ajeeta Singh (2021) Steady state behaviour of ATM with cold standby of bank computers having internal and external repair policy. *Life cycle Reliability and Safety Engineering* 10():161-171.
2. Ashok Kumar, D. Pawar & S.C Malik (2020) Reliability Analysis of a Redundant System with 'FCFS' Repair Policy Subject to Weather Conditions. *Int. J. of Advanced Science and Technology* 29(3): 7568-7578.
3. Barak M.S., Neeraj, and Kumari S (2018) Profit analysis of a two-unit cold standby system operating under different weather conditions subject to inspection. *Application and Applied Mathematics* 13(1):67-83.
4. Barzkar, A., Najafzadeh, M. & Homaei, F. (2022) Evaluation of drought events in various climatic conditions using data-driven models and a reliability-based probabilistic model. *Nat Hazards* 110():1931–1952.
5. C. Chen, J. Wang and D. Ton (2017) Modernizing Distribution System Restoration to Achieve Grid Resiliency Against Extreme Weather Events. *An Integrated Solution in Proceedings of the IEEE* 105(7):1267-1288.
6. C. Guo, C. Ye, Y. Ding and P. Wang (2020) A Multi-State Model for Transmission System Resilience Enhancement Against Short-Circuit Faults Caused by Extreme Weather Events in *IEEE Transactions on Power Delivery* 36(4): 2374-2385.
7. C. Wang, Y. Hou, Z. Qin, C. Peng and H. Zhou (2015) Dynamic Coordinated Condition-Based Maintenance for Multiple Components with External Conditions in *IEEE Transactions on Power Delivery* 30(5): 2362-2370.
8. Deswal S. and Malik S. C. (2015) Reliability measures of a system of two non-identical units with priority subject to weather conditions. *Journal of Reliability and Statistical Studies* 8(1): 181–190.
9. D. Chakraborty, S. D. Nair and M. Mukherjee (2023) Rain-Based Attenuation and Dispersion Characteristics of Terahertz Wave in Tropical Climate: Experimentally Verified Reliability Study in *IEEE Access* 11():36605-36617.
10. F. Mujjuni, T. R. Betts and R. E. Blanchard (2023) Evaluation of Power Systems Resilience to Extreme Weather Events: A Review of Methods and Assumptions, in *IEEE Access*, 11():87279-87296.
11. Gahlot, Monika; Singh, V.V.; Ayagi, Hamisu Ismial; Goel, C.K. (2018) Performance assessment of repairable system in series configuration under different types of failure and repair policies using copula linguistics. *International Journal of Reliability and Safety*, 12(4):348.

12. Hassan Iqbal, Solomon Tesfamariam, Husnain Haider & Rehan Sadiq (2017) Inspection and maintenance of oil & gas pipelines: a review of policies, *Structure and Infrastructure Engineering* 13(6):794-815.
13. Himani Sharma (2017) Reliability study of Solar PV Power Production In Terms Of Weather Parameters Using Monte Carlo Simulation, *International Journal of Engineering Research and Applications (IJERA)* 7(7) : 37-45.
14. Kumar, A., Saini, M. (2018) Comparison of reliability characteristics of two semi-Markov repairable systems under degradation and abnormal environment. *Life Cycle Reliab Saf Eng* 7(): 257–268.
15. Kumar A, Saini M (2016) Analysis of some reliability measures of single unit systems subject to abnormal environment conditions and arbitrary distribution of failure and repair activities. *J Inf Optim Sci* 39(2):545–559.
16. Koop, S.H.A., van Leeuwen, C.J. (2015) Assessment of the Sustainability of Water Resources Management: A Critical Review of the City Blueprint Approach. *Water Resour Manage* 29(): 5649–5670.
17. Kumar A, Saini M (2016) Impact of abnormal weather conditions on various reliability measures of a repairable system with inspection. *Thailand Stat* 14(1):35–45.
18. Puspendu Ghosh & Mala De (2022) A comprehensive survey of distribution system resilience to extreme weather events: concept, assessment, and enhancement strategies, *International Journal of Ambient Energy* 43(1): 6671-6693.
19. Saini, M., & Kumar, A. (2021) Stochastic Modelling and Sensitivity analysis of an integrated Hardware-Software system subject to different weather conditions. *Malaysian, Journal of Science* 40(3): 54–63.
20. Sonal, S. K. Sahu, D. Ghosh and D. K. Mohanta (2019) Reliability Analysis of a Distribution Network in Two State Weather Condition. *International Conference on Computer, Electrical & Communication Engineering (ICCECE)*, Kolkata, India ()():1-6.
21. Shan, Xiaofang; Wang, Peng; Lu, Weizhen (2017) the reliability and availability evaluation of repairable district heating networks under changeable external conditions, *Applied Energy*,203(),686-695.
22. S. Ma, B. Chen and Z. Wang (2018) Resilience Enhancement Strategy for Distribution Systems Under Extreme Weather Events, in *IEEE Transactions on Smart Grid* 9(2): 1442-1451.
23. Zamani Gargari, Milad; Ghaffarpour, Reza (2020) Reliability evaluation of multi-carrier energy system with different level of demands under various weather situation. *Energy*, 196(), 117091–.
24. Vijay Vir Singh, Praveen Kumar Poonia, Ameer Hassan Adbullahi (2020) Performance analysis of a complex repairable system with two subsystems in series configuration with an imperfect switch, *J. Math. Comput. Sci.*, 10(2): 359-383.
25. Y. Wu, T. Fan and T. Huang (2020) Electric Power Distribution System Reliability Evaluation Considering the Impact of Weather on Component Failure and Pre-Arranged Maintenance in *IEEE Access* 8():87800-87809.